Organizational Structure and Gray Markets

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April 9, 2012

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Abstract

Gray marketing, the selling of branded goods outside of manufacturer-authorized channels, is a factor in many industries. Using a model of differentiated Cournot competition, we analyze how gray markets affect the strategy used to enter low-priced foreign markets. In particular, we examine how much autonomy a firm should give its foreign subsidiary. When production in the foreign country is determined at head office, the firm has a centralized organizational structure. In contrast, when the subsidiary has autonomy to set its own production, the firm has a decentralized organizational structure. In the presence of gray markets, we find that organizational structure has a significant effect on firm profitability. When competing products in the domestic market are highly substitutable, foreign entry accompanied by decentralized management is advantageous. The finding holds in a situation where only one firm enters the foreign market but also in a situation where both firms do so. The advantage of decentralization is not explained by lower gray market volume under decentralization: gray market quantities can also be higher. The advantage of decentralized management comes from the different incentives it creates for decision makers. Under decentralization, every division makes production decisions locally. This leads to aggressive production in the domestic market because the decision maker does not account for reduced gray market sales (this "hurts" the foreign subsidiary). Aggressive domestic production both limits the impact of the gray market and weakens the domestic competitor. The same mechanism also applies when both firms enter the foreign market. As a result, both firms adopt decentralized control but in contrast to the single firm case, the equilibrium is a Prisoners’ Dilemma: profits are reduced versus an outcome where both firms operate under centralized control. The results suggest that in competitive categories where gray marketing is significant, the likelihood that foreign subsidiaries operate independently is higher. These results echo the findings of McGuire and Staelin (1983) who find that manufacturers in bilateral duopolies benefit by decentralizing operations when downstream products are close substitutes. However, contrary to the finding for bilateral duopolies, here manufacturers suffer as a result of their decentralized structure.

Keywords: gray markets, diversion, foreign market entry.
1 Introduction

1.1 Background

Gray marketing, also known as diversion or parallel importing, is the selling of genuine branded goods outside of manufacturer-authorized channels. Whenever genuine branded goods are available for sale at different prices in different markets or channels, the potential for gray marketing exists.\(^1\) Manufacturers in high-priced markets like the US often encounter significant quantities of gray market goods that have been imported to the US from markets where price levels are lower. In a recent high-profile example, Costco purchased genuine Omega watches overseas for substantially less than Omega’s US retail price, and then sold these watches in competition with Omega’s authorized channel.\(^2\)

Gray marketing affects a wide range of categories: common examples include clothing, IT products, luxury goods, portable phones, soft drinks and pharmaceutical products. While the importance of gray marketing varies by category, the impact is large. A 2009 analysis by Deloitte LLP estimates that gray market sales cost manufacturers collectively as much as $63 billion in annual sales.\(^3\) The Alliance for Gray Market and Counterfeit Abatement estimated in 2008 that the value of gray market products in the information technology (IT) sector alone was $58 billion (8% of global sales) annually, representing lost profits of up to $10 billion.\(^4\) Gray market sales of pharmaceuticals in the European Union are estimated to comprise nearly 20% of the UK market and 10% or more of the Dutch, Danish and Swedish markets, while gray market "exports" (outflow) represented 22% of the Greek market (Kanavos and Costa-Font 2005). In the cell phone industry, Fortune reported that in 2007 as many as 1 million iPhones (out of 3.75 million sold) were estimated to have been diverted to the gray market.\(^5\)

In general, there is little manufacturers can do to stop this activity: high court rulings in the United States and Europe have upheld the legality of gray market sales as being in the best interest of consumers.\(^6\)

\(^1\)Although we frame this paper as gray market diversion from a foreign to a domestic market, the model is not restricted to geographically separate markets. It equally applies to a situation where a branded manufacturer attempts to price discriminate across different markets or channels and leakage across the markets or channels is prevalent.


\(^3\)Ibid.

\(^4\)Refer to www.agmaglobal.org.

\(^5\)#Apple’s $300 million gray market dilemma” (see tech.fortune.cnn.com/2008/01/28/apples-300-million-gray-market-dilemma/)
of consumers. In the US, the standing Supreme Court precedent ruled that the "first sale doctrine" (17 U.S.C. §109(a)) provides a defense to copyright infringement. Thus, companies which legally purchase goods abroad that were intended for foreign consumers can legally resell those goods in the U.S. despite objections of the brand owner.\footnote{Because the US Supreme Court deadlocked in the Costco v. Omega case, the relevant US legal precedent is \textit{Quality King Distributors, Inc. v. L'Anza Research International Inc.} 523 U.S. 135 (1998). For legal details, see Aldridge (2011).} In the European Union, the European Court of Justice (ECJ) has similarly applied the analogous "exhaustion of rights doctrine" (Treaty of Rome, article 30).\footnote{Kanavos and Costa-Font (2005) detail the series of ECJ rulings and opinions that underpin the legitimacy of the gray market ("parallel trade") in the European Union.} Since manufacturers cannot stop gray markets, the key issue is how best to manage them: a critical consideration for a firm when it contemplates entry to a lower priced foreign market is how gray market activity affects overall profitability.

The first question a manufacturer needs to answer is "does it make sense to enter a low-priced foreign market given the sales that gray markets may cannibalize domestically?" However, given that foreign entry may be both profitable and a key element of a firm’s global strategy, a second question is how to best enter the foreign market (given the inevitability of diversion). One alternative is to give local managers autonomy to set foreign production levels to maximize the foreign subsidiary’s performance (a decentralized organizational structure). A second alternative is that the firm makes production decisions for the foreign market at the domestic head office (a centralized organizational structure).

The issue of organizational structure is the main question we consider. On the one hand, giving local managers autonomy optimizes local performance. When markets are independent, this autonomy leads to optimal decision making. On the other hand, markets are not independent when there are gray markets. Rather, the gray market is an important externality between the foreign market and the domestic market. The literature suggests that an effective strategy to optimize in the face of an externality is to centralize decision making (Varian 1992). Moreover, recent research (Antia, Bergen, Dutta and Fisher 2006) advocates centralized decision making to minimize the negative impact of gray market activity. While conventional wisdom suggests that centralization should be effective to internalize the gray market externality, the optimal strategy when faced with a competitor who has similar challenges is not clear.

Accordingly, we construct a model of differentiated product competition to better understand the impact of gray marketing on a firm’s strategic decisions. We consider both the strategic decisions of whether to enter a lower-priced foreign market and, if so, what organizational structure a firm
should establish. We show that organizational structure is a key determinant of performance for a firm that faces both competition and gray market goods in its domestic market. In addition, we demonstrate the robustness of these findings by extending the analysis to a) a situation where the gray marketer is modelled as a competitive fringe and b) a context where firms engage in price competition after making decisions about capacity.

1.2 Literature Review

As noted in Kotler and Keller (2009), many firms use multi-channel marketing due to the proliferation of both customer segments and channels. A polar case of the phenomenon is the desire of Western firms to enter new markets through the establishment of foreign subsidiaries. The driving force behind these new channels is the opportunity to serve new customers profitably. Of course, there are costs to adding channels beyond the cost of simply managing and dealing with another customer. As noted by Coughlan, Anderson, Stern, and El-Ansary (2006), these costs include “conflict” that may occur when these channels compete for the same customers. This is particularly salient when entrepreneurial gray market firms divert product from lower priced developing markets back to the domestic market.

Many academics have analyzed the topic of gray markets to better understand their overall effect on an industry. Antia et al. (2006), Assmus and Wiese (1995), Weigand (1991), Cespedes, Corey and Rangan (1988) and Cavusgil and Sikora (1988) take the position that gray markets are a problem for manufacturers for reasons that include losing control of distribution, a decreased ability to price discriminate, and the erosion of brand equity. There is also a stream of literature that examines the impact of gray markets on performance at the firm level. For example, Li and Robles (2007) suggest that gray markets may generate unanticipated consequences for firms by stifling multinationals’ incentives to invest in research and development. While firms often suffer as a result of gray marketing, Maskus and Chen (2004) and Autrey and Bova (2012) show that global surplus is often increased by gray market activity. This may explain why the courts are reluctant to rule against firms that facilitate the availability of gray market goods. In any event, diversion creates a complexity that firms need to manage. Banerji (1990) highlights the challenge of price-discrimination for a monopolist in this context. Surprisingly, he finds little effort by IBM to stop authorized IBM resellers from supplying unauthorized gray marketers. Perhaps gray markets are not as serious a problem for branded manufacturers as assumed in the popular press.

Interestingly, a few studies identify situations in which gray markets do not generate a negative
externality for manufacturers. Bucklin (1993) suggests that price erosion in the home market is frequently offset by an increase in unit sales, while Ahmadi and Yang (2000) suggest that gray markets might extend the firm’s global reach and improve global profits. Raff and Schmitt (2007) demonstrate that letting retailers trade unsold inventories to the gray market increases retailer orders given demand uncertainty and can lead to higher manufacturer profit, and Chen (2009) shows conditions under which gray markets may help a firm segment its home market via the service level selected by authorized retailers.

A limitation of many gray market studies is that they do not consider the impact of competition on optimal strategies. The standard approach is to examine the challenge of a monopolist who loses the ability to price discriminate as a result of gray markets. Gallini and Hollis (1999) highlight the motivation that a firm has to prevent or limit gray marketing. Not surprisingly, significant research is dedicated to analyzing the alternatives manufacturers have to limit the impact and magnitude of gray markets. There are also scores of articles in the business press which highlight the negative impact of gray markets and provide guidance on how gray marketing can be minimized (for a typical example see Dove and Hamilton 2008).

A frequently mentioned alternative to reduce the negative effect of gray market activity is for a manufacturer to centralize decision making at head office. After all, the presence of gray market goods invariably requires a source and this source typically does not act in the best interest of the manufacturer. In many (if not all) cases, the source of gray market goods is a foreign subsidiary (or a third party) which has the rights to market the product in another territory.

The role of centralization as a strategy to control foreign subsidiaries is well known (Gatignon and Anderson 1988). It follows that several papers (e.g., Antia et al. 2006, Assmus and Wiese 1995) propose centralized decision making as a strategy to reduce the negative impact of gray markets. Examples of centralization include product rationing to foreign subsidiaries or uniform pricing across international jurisdictions to eliminate the gray market arbitrage opportunities. There is also empirical evidence to support the idea that centralization is effective to counter gray marketing. For example, Myers (1999) surveys a sample of U.S. multinationals and finds that the greater the degree of centralized decision making, the lower the reported incidents of gray market activity. Interestingly, centralized control has also been suggested as a strategy "to keep unauthorized reselling to a minimum" (Doyle 1997). Our objective is to challenge this conventional wisdom and to assess whether and when centralization can be effective to minimize the negative impact of gray markets.

Our approach is to study a context where the manufacturer faces a choice of imposing centralized
control on a foreign subsidiary or allowing the foreign subsidiary to operate independently. A key objective of the analysis is to examine this question in an environment where two manufacturers compete with each other. This allows us to assess both the direct effect and the indirect effect of gray market goods on a focal manufacturer’s profits. Most extant models consider the direct effect of gray market goods on the manufacturer’s profits, i.e., the cannibalization of demand for the manufacturer’s product in the domestic market. However, our model will also consider the indirect effect of gray market goods on the manufacturer’s global profit, i.e., the gray market goods of one manufacturer affect the behavior of the existing competitor. This too affects the performance of the focal manufacturer, and the model reflects this.

1.3 Summary of Key Findings

Our analysis shows that organizational structure has a significant effect on firm performance when markets are competitive and gray marketers are active.

It is important to note that under decentralization, each subsidiary sets quantity to maximize local profits: a subsidiary does not internalize the fact that its decisions affect the profit earned by the other subsidiary. To be precise, the foreign subsidiary neglects the cannibalization of domestic sales by the gray market product, and the domestic subsidiary ignores the effect that its volume decisions have on gray market demand.

Our model shows that when competing products in the domestic market are highly substitutable, a firm increases its global profit through the adoption of a decentralized structure even though local managers only consider local profit performance when making decisions. This finding holds in a situation where only one firm has the capability to expand in the foreign market but also in a situation where both firms have the capability. Additionally, this finding is not explained by higher gray market volume under decentralization. In fact, with decentralization, gray market quantities can be either higher or lower than the quantities observed under centralized control.

Rather, decentralization plays a subtle role in improving the focal firm’s profitability. Specifically, the domestic branch of the firm makes its decisions based on local profit performance when a decentralized organizational structure is implemented. This leads to a domestic branch that is more aggressive with its production decisions: it does not account for lower gray market sales (by the foreign subsidiary). The strategic effect of this is to both limit the negative effect of the gray market and weaken the existing domestic competitor.

In a nutshell, when only one firm enters the foreign market (say Firm 1), decentralization
makes the domestic market more competitive than under centralization. This reduces the size of the domestic profit pie. However, as product substitutability increases, decentralization increases the proportion of total domestic profits that accrue to Firm 1 compared to centralization. Thus, decentralization induces a tension between creating lower overall profits for all domestic firms through increased competition, and increasing Firm 1’s proportion of those profits. When the products of Firms 1 and 2 are sufficiently similar, the increase in the proportion of total domestic profits that accrue to Firm 1 under decentralization more than offsets the reduced size of the domestic pie created by decentralization, leaving Firm 1’s domestic arm better off. When this effect is sufficiently strong, decentralization leads to higher global profits for Firm 1. This finding seems to belie conventional wisdom: centralization typically imposes managerial costs on an organization but it is generally associated with higher profit potential. Here though, the strategic interaction of channel players is important and decentralization leads to higher profits because it weakens the existing domestic competitor "indirectly".

In contrast, when both firms can enter the foreign market and the products are highly substitutable, the firms find themselves in a Prisoners’ Dilemma. In this situation, the more aggressive stance of the domestic subsidiary makes decentralization the optimal response to a firm that has a centralized structure. In addition, the optimal response to entry with a decentralized structure is to also adopt a decentralized structure. The more aggressive stances of the subsidiaries in both the domestic and foreign markets however, result in lower equilibrium profits for both firms. Taken together, we have a general finding that a decentralized organizational structure is optimal for firms entering foreign markets when gray markets are active and the degree of substitutability between products is high. However, the effect of organizational structure on profits differs depending on whether only one firm or both firms have the capability to enter the foreign market.

When the degree of substitutability between the domestic competitors is high and only one firm can enter the foreign market, it realizes greater profits as a result of decentralizing. In contrast, when both firms can enter the foreign market, decentralization is the equilibrium organizational structure yet this outcome is associated with lower profits for both firms. In a sense, the advantage that leads to the dominance of decentralization when only one firm enters the foreign market is precisely what leads to more intense competition and lower profits when both firms enter with a decentralized structure. The optimality of "decentralization" when products are highly substitutable appears

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When the degree of substitutability is moderate, decentralization is not unique. For an intermediate range of substitutability, there are multiple equilibria including a mixed strategy equilibrium and a pure strategy equilibrium of centralization (for both firms).
to echo the findings of McGuire and Staelin (1983) who study the effect of decentralization in a bilateral duopoly. However, in contrast to McGuire and Staelin where both manufacturers benefit from decentralization as products become more substitutable, here both manufacturers suffer as a result of their inability to commit to a centralized structure.

2 Model Setup

We consider a setting where two firms engage in quantity competition in a domestic market with differentiated products. Each firm may also sell its product in a foreign country (where competitive prices are lower) through a wholly-owned foreign subsidiary.\(^9\) If either firm enters the foreign market, a gray market firm might purchase some of the firm’s goods in the foreign market and resell those goods in the domestic market.\(^{10}\) For simplicity, we assume that the product sold in the foreign market by each firm is identical to its product in the domestic market.

Interestingly, if a multinational firm allowed a foreign subsidiary to operate domestically by vertically integrating gray market activity (for example), it would earn strictly higher profits.\(^{11}\) However, this does not happen for two reasons. First, there might be a backlash when customers realize that two subsidiaries of a firm are supplying the same product at different prices. But most importantly, this type of distribution is prohibited by competition or anti-trust law. Though we do not explicitly model retailers, domestically produced goods and gray goods are distributed through retailers. The law obliges any firm to sell its product at the same price to retailers who compete in the same market.\(^{12}\) Allowing a foreign subsidiary to sell the same product at a different price would violate this regulation (since a foreign subsidiary is \textit{de jure} the same company).

We consider two competitive situations in the model. The first is a situation where only one domestic competitor has the ability to enter the foreign market. In the second situation, both firms have the ability to enter the foreign market. In addition to making the entry decision, a firm that decides to enter also makes a decision about organizational structure. The model considers two organizational options. First, a firm can choose to control both the domestic and foreign

\(^9\) For simplicity, we examine a model with a domestic market with higher price levels and one foreign market with lower price levels. However, as correctly pointed out by an anonymous reviewer, the logic of the model applies to the diversion of product from any number of lower priced markets back to a higher priced domestic market.

\(^{10}\) We assume that domestic consumers are not capable of purchasing goods in the foreign market. Relative to consumers, the gray marketer has specialized knowledge where to find appropriate goods and has expertise in transshipping. Gray markets exist precisely because the per-unit transaction costs of domestic consumers to acquire foreign goods are significantly higher than those of the gray marketer.

\(^{11}\) We thank an anonymous reviewer for having made this insightful observation.

\(^{12}\) In Canada, the relevant law is Price Discrimination Enforcement Guidelines section 50 (1) (a) of the Competition Act. In the US, the relevant law is the Robinson–Patman Act of 1936, section 2a.
subsidiary centrally. In this setting, both production decisions are made by managers in the head office. Such a structure implies that production quantities are set to account for the impact of each decision on total firm profit (centralized control). Alternately, a firm can choose to decentralize and give the domestic and foreign subsidiary profit responsibility and control of their respective production decisions (decentralized control). A decentralized structure implies that the manager in each country makes decisions to optimize local profits.\footnote{Note that our model setup differs from that of McGuire and Staelin (and similar setups in Arya, Mittendorf, and Yoon 2008). The McGuire and Staelin model assesses the optimal distribution choices of two competing manufacturers. Each manufacturer can either distribute through an independent exclusive retailer or it can "vertically integrate" the retailer and distribute directly to end consumers.}

We assume that the product’s marginal cost is constant and (without loss of generality) zero, and profits in the domestic and foreign market are of equal value. Without loss of generality, we further assume that the cost of shipping gray marketed product back to the domestic market is negligible. The model thus focuses on a situation where the effects of the gray market are as high as possible to understand how it can be managed effectively.

We begin with a model of Cournot quantity competition, in which the two rival firms (the Stackelberg leaders) choose quantity first, anticipating the subsequent quantity choice by the gray marketer (the Stackelberg follower). We structure our analysis as a game of quantity competition for two reasons. First, many authors use quantity competition to model gray markets. Recent papers include Li and Robles (2007), Chen (2009) and Autrey and Bova (2012). This suggests that quantity competition is appropriate to represent how firms compete in a gray market setting. Second, there are many accounts to indicate that gray marketers make decisions related to the quantity that is trans-shipped and not price. For example, the gray market for cigarettes in the provinces of Ontario and Quebec is substantial due to the existence of First Nations reserves that either straddle or are close to the U.S.-Canada border. A report by Luk, Cohen and Ferrence (2007) provides a complete description of how the gray market for cigarettes operates. Specifically, gray marketers do not set prices for their goods. Instead, they buy quantities of goods in one market based on their belief about the prices that they can obtain for the goods in the higher priced market. There is no guarantee of prices before the goods are acquired and offered for resale in the higher priced market. If the gray marketer buys too much product, then it is faced with a problem of trying to liquidate stock and this reduces profitability.

For these reasons, we feel that the Cournot model is an appropriate representation of the economy when a gray market firm is present. For robustness however, we present two alternative
models of competition in Section 3.5. In the first alternative, we model the gray market as a
competitive fringe player. In the fringe model, the gray marketer has no market power and is a
price taker in the domestic market. In the second alternative, we present a model of Bertrand
price competition with capacity constraints. In this setting, the two firms first choose manufacturing
capacity before engaging in price competition. The main model results are robust to either of these
alternative formulations.

2.1 Firm organizational decision

The first decision made by firms is whether to: a) enter the foreign market with a decentralized
structure, b) enter the foreign market with a centralized structure or c) not enter the foreign mar-
ket. Firms make this decision knowing that the gray marketer may divert a fraction of foreign
production to the domestic market. Under a decentralized structure, the individual country subs-
idiaries maximize local profit in their respective markets. In a centralized structure, the central
headquarters maximizes overall firm profit. We denote the strategy set for both Firm 1 and Firm
2 in the first stage of the game as \( s_i = \{N = \text{no entry}, D = \text{entry with decentralized control}, C = \text{entry with centralized control}\} , i = 1, 2 \). While the potential sales in the foreign market are
invariably attractive, the negative effect of diversion needs to be accounted for in making this de-
cision. Once the entry decisions and the organizational structures have been chosen, Firms 1 and
2 compete in their markets as detailed below.

2.2 Demand structure and timeline

After choosing their organizational structure, Firms 1 and 2 choose quantities and the market
demand determines the resulting price. The domestic demand for each firm is as follows:

\[
p_i = 1 - (q_i + q_{Gi}) - \gamma(q_j + q_{Gj}), \quad i, j = 1, 2, i \neq j
\]

where \( q_i \geq 0 \) is the quantity sold at price \( p_i \) by Firm \( i \) in the domestic market, \( q_{Gi} \geq 0 \) is the
gray market quantity diverted from Firm \( i \)'s foreign market, and \( \gamma \in [0, 1] \) represents the degree of
substitutability between the products of Firms 1 and 2.

The foreign subsidiaries face the following demand functions, where \( F \in (0, 1) \):

\[
p_{Fi} = F - q_{Fi} - \gamma q_{Fj}, \quad i, j = 1, 2, i \neq j
\]

where \( q_{Fi} \geq 0 \) is the quantity sold at price \( p_{Fi} \) by Firm \( i \) in the foreign market.
The gray marketer’s demand for each firm’s product, \( q_{Gi} \), is determined endogenously to maximize profit and is incremental to the demand that each foreign subsidiary obtains from local customers, \( q_{Fi} \). We assume that the gray marketer diverts product from the foreign market and sells it in the domestic market. Thus, the gray marketer’s profit per unit is \( p_i - p_{Fi} \), the price differential observed between the domestic and foreign markets.\(^{14}\) Note that if the price in the foreign market is sufficiently high, the gray market will not function. Alternatively, if the equilibrium price in the foreign market is sufficiently low, domestic firms will not enter the foreign market.\(^{15}\)

Given the anticipated gray market activity, the entry decisions and the organizational structures that have been chosen, Firms 1 and 2 (the Stackelberg leaders) choose quantities (local and foreign, where applicable) to maximize their profits. Next, the gray marketer (the Stackelberg follower) assesses the profitability of acquiring a quantity of product in the foreign market and reselling that same quantity in the domestic market. In fact, we assume that the gray marketer acquires a "profit-maximizing" quantity in the foreign market and resells it in the domestic market at the market clearing price. Of course, when the gray market is active, the volume sold in the domestic market is higher and naturally, this leads to a lower equilibrium domestic price.

Consistent with standard Cournot models, product prices in the domestic and foreign markets (respectively) are determined such that the market clears (Mas-Colell, Whinston and Green 1995). This game structure assumes that a) foreign consumers and the gray marketer pay the same price for product purchased in the foreign market and b) the gray marketer and the respective domestic firm each receive the same price for product sold in the domestic market. The timing of the decisions in the game is shown in Figure 1.

2.2.1 Decentralized Decision Making (only one firm can enter)

Without loss of generality, we assume that Firm 1 is the player that has the capability to enter the foreign market. If \( Q \) is the total production of Firm 1’s foreign subsidiary, the objective functions for Firm 1 are:

\[
\begin{align*}
\pi_1 &= p_1 q_1 \\
\pi_F &= p_F Q
\end{align*}
\]

\(^{14}\)The model is robust to restricting the gray market’s selling price to some fraction \( \beta < 1 \) of the authorized channel’s price, or \( \beta p_i \) instead of \( p_i \). As pointed out by a reviewer, this reflects a situation in which gray market goods are sold at a discount relative to domestically produced goods. With this alternative specification, the results are identical, although the smaller is \( \beta \), the less important controlling the gray market becomes. For simplicity, we omit the \( \beta \) parameter.

\(^{15}\)When the price in the foreign market is sufficiently low, gray market goods collapse profits in the domestic market and the only beneficiary of foreign entry is the gray marketer.
1) Manufacturer(s) make entry decision (N, D or C)
2) Manufacturers simultaneously set domestic quantities and foreign quantities (where applicable)
3) Gray marketer chooses quantity(s) given demand conditions and the quantities chosen by manufacturers.
4) The market clears

Figure 1: Timeline

The objective function for Firm 2 is:

\[ \pi_2 = p_2 q_2 \] (4)

and for the gray marketer:

\[ \pi_G = (p_1 - p_F) q_G \] (5)

subject to the following constraints:

\[ p_1 = 1 - q_1 - \gamma q_2 - q_G \] and \[ p_2 = 1 - q_2 - \gamma (q_1 + q_G) \] (6)

\[ p_F = F - q_F \] and \[ Q = q_F + q_G \] (7)

Following the standard approach to solve Stackelberg games, we solve the gray marketer’s problem first and this generates a reaction function for \( q_G \) as a function of the quantities produced by the manufacturers in the first stage. This is then substituted into the objective functions for Firm 1’s domestic subsidiary, Firm 1’s foreign subsidiary and Firm 2’s objective function. These functions are optimized with respect to \( q_1 \), \( q_2 \) and \( q_F \) respectively to create a system of three equations in three unknowns.

2.2.2 Decentralized Decision Making (both firms can enter)

The objective functions for Firm 1 are:

\[ \pi_1 = p_1 q_1 \text{ and } \pi_{F1} = p_{F1} Q_1 \] (8)

The objective functions for Firm 2 are:

\[ \pi_2 = p_2 q_2 \text{ and } \pi_{F2} = p_{F2} Q_2 \] (9)
and for the gray marketer:

\[ \pi_G = (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2} \]  

(10)

subject to the following constraints:

\[ p_1 = 1 - q_1 - \gamma q_2 - q_{G1} - \gamma q_{G2} \text{ and } p_2 = 1 - q_2 - \gamma q_1 - q_{G2} - \gamma q_{G1} \]  

(11)

\[ p_{F1} = F - q_{F1} - \gamma q_{F2} \text{ and } p_{F2} = F - q_{F2} - \gamma q_{F1} \]  

(12)

\[ Q_1 = q_{F1} + q_{G1} \text{ and } Q_2 = q_{F2} + q_{G2} \]  

(13)

As before, the gray marketer’s problem is solved first and this generates reaction functions for \( q_{G1} \) and \( q_{G2} \) as a function of the quantities produced by the manufacturers in the first stage. These are then substituted into the objective functions of Firms 1 and 2. These objective functions are optimized with respect to \( q_1, q_{F1}, q_2 \) and \( q_{F2} \) respectively to create a system of four equations in four unknowns.

2.2.3 Centralized Decision Making (only one firm can enter)

As before, if \( Q \) is the total production of Firm 1’s foreign subsidiary, the objective function for Firm 1 is:

\[ \Pi_1 = p_1 q_1 + p_{F1} Q \]  

(14)

The objective function for Firm 2 is:

\[ \pi_2 = p_2 q_2 \]  

(15)

and for the gray marketer:

\[ \pi_G = (p_1 - p_{F}) q_G \]  

(16)

subject to (6) and (7). Note that the constraints (i.e. the demand functions) are unaffected by the organizational structure of the firms.

The gray marketer’s problem is solved first and this generates a reaction function for \( q_G \) as a function of the quantities produced by the manufacturers in the first stage. This is then substituted into the objective functions of Firms 1 and 2. In contrast to the decentralized case, Firm 1 has one objective function and it is optimized with respect to \( q_1 \) and \( q_F \) simultaneously. This generates a system of three equations in three unknowns.
2.2.4 Centralized Decision Making (both firms can enter)

The objective function for Firm 1 is:

\[ \Pi_1 = p_1 q_1 + p_{F1} Q_1 \]  

(17)

The objective function for Firm 2 is:

\[ \Pi_2 = p_2 q_2 + p_{F2} Q_2 \]  

(18)

and for the gray marketer:

\[ \pi_G = (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2} \]  

(19)

subject to (11), (12), and (13).

As before, the gray marketer’s problem is solved first and this generates reaction functions for \( q_{G1} \) and \( q_{G2} \) as a function of the quantities produced by the manufacturers in the first stage. These are then substituted into the objective functions of Firms 1 and 2. These functions are then optimized with respect to \( q_1, q_{F1}, q_2 \) and \( q_{F2} \) respectively to create a system of four equations in four unknowns.

2.2.5 Asymmetric Decision Making (both firms enter)

Here, we assume that Firm 1 operates with a centralized structure and Firm 2 operates with a decentralized structure and both firms enter the foreign market. The objective function for Firm 1 is:

\[ \Pi_1 = p_1 q_1 + p_{F1} Q_1 \]  

(20)

The objective functions for Firm 2 are:

\[ \pi_2 = p_2 q_2 \] \text{ and } \[ \pi_{F2} = p_{F2} Q_2 \]  

(21)

and for the gray marketer:

\[ \pi_G = (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2} \]  

(22)

subject to (11), (12), and (13).

To solve the model where the firms are asymmetric, the gray marketer’s reaction functions for \( q_{G1} \) and \( q_{G2} \) as a function of the quantities produced by the manufacturers are substituted into the objective functions of Firms 1 and 2. These functions are then optimized with respect to \( q_1, q_{F1}, q_2 \) and \( q_{F2} \) respectively to create a system of four equations in four unknowns.
3 Model Analysis

We begin by deriving each firm’s optimal quantity choice and resulting profits under the possible organizational structures. For each structure, we first analyze the simple setting when only Firm 1 can enter the foreign market and then present the results when both firms can enter. We then analyze the resulting equilibria and discuss the implications of our results.

For each of the settings, we restrict all quantities to be non-negative. Domestic quantity is always non-negative because \( \gamma \in [0, 1] \) and \( F < 1 \). However, the gray market is only non-negative when \( F \) is sufficiently low. We define \( m_1(\gamma) \) and \( n_1(\gamma) \) as the upper bound of \( F \) for which \( q_{Gi} \geq 0 \) with a decentralized and centralized structure respectively. We also require \( q_{Fi} \geq 0 \); \( F \) must be sufficiently large that the optimal quantity in the foreign market is non-negative.\(^{16}\)

It should be noted that the equilibrium profit for the firms when neither enters the foreign market is \( \Pi = \frac{1}{(\gamma+2)^2} \). Therefore, a decision to enter the foreign market by one of the firms (given that the competitor does not have foreign operations) must yield an increase versus this level of profits.

3.1 Decentralized Structure

3.1.1 Only Firm 1 can enter the foreign market (Decentralized)

Solving the first order conditions presented in the appendix, we derive the following solution in terms of quantities for the decentralized case when only one firm enters the foreign market. For brevity, we do not present the first order conditions and move directly to the optimal quantities.

\[
q_1 = \frac{1}{32 \gamma^2 - 52} \left( 14 \gamma - 8F + 7 \gamma^2 + 6F \gamma^2 - 28 \right)
\]

\[
q_2 = \frac{1}{32 \gamma^2 - 52} \left( 19 \gamma - 2F \gamma - 26 \right)
\]

\[
q_F = \frac{1}{32 \gamma^2 - 52} \left( 22F \gamma^2 - 2 \gamma - \gamma^2 - 36F + 4 \right)
\]

\[
q_G = \frac{1}{32 \gamma^2 - 52} \left( 5 \gamma + 12F + \frac{5}{2} \gamma^2 - 7F \gamma^2 - 10 \right)
\]

(23)

These quantities imply the following profits for the three players (Firm 1’s profit is the sum of its domestic profit and foreign profit).

\[
\Pi_1 = \frac{(14 \gamma - 8F + 7 \gamma^2 + 6F \gamma^2 - 28)^2 + 3 (2 \gamma - 16F + \gamma^2 + 10F \gamma^2 - 4)^2}{32 (8 \gamma^2 - 13)^2}
\]

(24)

\[
\pi_2 = \frac{(2F \gamma - 19 \gamma + 26)^2 (2 - \gamma^2)}{32 (8 \gamma^2 - 13)^2}
\]

(25)

\(^{16}\)This constraint never binds with centralized entry, and only binds for a small range of parameter values when both firms choose decentralized entry (in particular, when \( \gamma \) is very close to 1 and \( F \) is very low). We define \( d(\gamma) \) as the lower bound of \( F \) such that \( F > d(\gamma) \) ensures \( q_{Fi} \geq 0 \) (shown in Figure A3 in the Appendix).
We define $f_1(\gamma)$ as the lower bound of $F$ for which Firm 1 enters the foreign market with a decentralized structure. A small $F$ has two effects on Firm 1’s entry decision. First, there is lower profit potential from entering the foreign market. Second, the gray market’s cost base is correspondingly lower and the gray market cannibalizes more of Firm 1’s domestic sales. For $F$ sufficiently low, Firm 1 does not enter the market because the loss of domestic sales to the gray market more than offsets the profit potential of entering the foreign market. The expression for $f_1(\gamma)$ is provided in the Appendix. Figure 2 shows the parameter region where Firm 1 will enter the market (above $f_1(\gamma)$, the solid line). Figure 2 also illustrates the infeasible region above the cutoff $m_1(\gamma)$, where the gray market quantity is zero. In this region, the market price in the foreign market is higher than in the domestic market. Thought of another way, this is theoretically a region where the gray market would flow in the opposite direction, from the domestic market to the foreign market. Because the focus of our analysis is to understand the challenge of a domestic firm entering a low-priced foreign market where a gray marketer funnels product back to the domestic market, we focus our attention on the feasible region between $f_1(\gamma)$ and $m_1(\gamma)$.
3.1.2 Both Firms can enter the foreign market (Decentralized)

Solving the first order conditions presented in the appendix, we derive the following solution in terms of quantities for the decentralized case when both firms enter the foreign market.

\[ q_1 = q_2 = \frac{2F + 3\gamma + 7}{13\gamma + 3\gamma^2 + 13} \quad \text{and} \quad q_{F1} = q_{F2} = \frac{9F + 11F\gamma + 3F\gamma^2 - 1}{26\gamma + 16\gamma^2 + 3\gamma^3 + 13} \quad (27) \]

\[ q_{G1} = q_{G2} = \frac{5 - 6F + 3\gamma - 4F\gamma}{2(\gamma + 1)(13\gamma + 3\gamma^2 + 13)} \quad (28) \]

These quantities imply the following profits for the three players.

\[ \Pi_1 = \Pi_2 = \frac{(2F + 3\gamma + 7)^2 + 3(4F + 2F\gamma + 1)^2}{2(13\gamma + 3\gamma^2 + 13)^2} \quad \text{and} \quad \pi_g = \frac{(6F - 3\gamma + 4F\gamma - 5)^2}{2(\gamma + 1)(13\gamma + 3\gamma^2 + 13)^2} \quad (29) \]

3.2 Centralized Structure

3.2.1 Only Firm 1 can enter the foreign market (Centralized)

Solving the first order conditions presented in the appendix, the unique solution in terms of quantities for the centralized case when only one firm enters the foreign market is:

\[ q_1 = \frac{1}{11\gamma^2 - 2\gamma^2 - 12} (6\gamma + 3\gamma^2 + F\gamma^2 - 12) \]
\[ q_2 = \frac{1}{11\gamma^2 - 2\gamma^2 - 12} (4\gamma - F\gamma - 6) \]
\[ q_F = \frac{1}{13\gamma^2 - 24} (9F\gamma^2 - 2\gamma^2 - 16F + 4) \]
\[ q_G = \frac{1}{11\gamma^2 - 2\gamma^2 - 12} (2\gamma^2 - 2F\gamma^2 + 4F - 4) \quad (30) \]

These imply the following profits for the three players.

\[ \Pi_1 = \frac{48F - 48\gamma - 24F\gamma + 48F^2 - 12\gamma^2 + 12\gamma^3 + 3\gamma^4}{2(7\gamma^2 - 12)^2} \]
\[ \pi_2 = \frac{(F\gamma - 4\gamma + 6)^2 (2 - \gamma^2)}{2(7\gamma^2 - 12)^2}, \pi_g = \frac{(2F\gamma^2 - 2\gamma^2 - 4F + 4)^2}{4(7\gamma^2 - 12)^2} \quad (31) \]

We define \( g_1(\gamma) \) as the lower bound of \( F \) for which Firm 1 enters the foreign market with a centralized structure. Similar to the decentralized case, when \( F \) is sufficiently low, Firm 1 does not enter the market because the loss of domestic sales to the gray market more than offsets the profit potential of entering the foreign market.\(^{17}\) The expression for \( g_1(\gamma) \) is provided in the Appendix. The feasible zone for gray marketing in the case of a centralized structure is shown in Figure 3. Figure 3 also illustrates the infeasible region above the cutoff \( n_1(\gamma) \), where the gray market quantity violates the non-negativity constraint under a centralized organizational structure. As in the decentralized case, the market price in the foreign market is higher than in the domestic market in this region.

\(^{17}\) A firm also earns profit on gray market volume but the prices (or the absolute margin) on gray market volume are significantly lower than prices earned in the domestic market.
3.2.2 Both Firms can enter the foreign market (Centralized)

Solving the first order conditions presented in the appendix, we derive the following solution in terms of quantities for the centralized case when both firms establish foreign subsidiaries.

\[ q_1 = q_2 = \frac{1}{\gamma + 2} \quad \text{and} \quad q_{F1} = q_{F2} = \frac{4F + 3F\gamma - 1}{9\gamma + 3\gamma^2 + 6} \quad \text{and} \quad q_{G1} = q_{G2} = \frac{1 - F}{9\gamma + 3\gamma^2 + 6} \quad (32) \]

These imply the following profits for the three players.

\[ \Pi_1 = \Pi_2 = \frac{2}{3} \left( \frac{F + F^2 + 1}{(\gamma + 2)^2} \right) \quad \text{and} \quad \pi_g = \frac{2}{9} \left( \frac{(1 - F)^2}{(\gamma + 2)^2 (\gamma + 1)} \right) \quad (33) \]

3.3 Asymmetric Structure

Solving the first order conditions presented in the appendix, we derive the solution in terms of quantities for the asymmetric case when Firm 1 is centralized and Firm 2 is decentralized:

\[ q_1 = \frac{3\gamma^3 - 4F\gamma - 6\gamma^2 - 14\gamma + 2F\gamma^2 + 26}{3\gamma^4 - 26\gamma^2 + 52}, \quad q_2 = \frac{8F - 14\gamma - 4F\gamma - 6\gamma^2 + 3\gamma^3 + 28}{3\gamma^4 - 26\gamma^2 + 52} \quad (34) \]

\[ q_{F1} = \frac{104F + 6\gamma + 24F\gamma + 3\gamma^2 - 66F\gamma^2 - 6F\gamma^3 + 9F\gamma^4 - 26}{3(\gamma + 1)(3\gamma^4 - 26\gamma^2 + 52)} \quad (35) \]

\[ q_{F2} = \frac{108F + 20\gamma + 28F\gamma - 3\gamma^3 - 66F\gamma^2 - 6F\gamma^3 + 9F\gamma^4 - 12}{3(\gamma + 1)(3\gamma^4 - 26\gamma^2 + 52)} \quad (36) \]

\[ q_{G1} = \frac{52 - 52F - 30\gamma + 36F\gamma - 15\gamma^2 + 18F\gamma^2 + 9\gamma^3 - 12F\gamma^3}{6(\gamma + 1)(3\gamma^4 - 26\gamma^2 + 52)} \quad (37) \]

\[ q_{G2} = \frac{30 - 36F - 11\gamma + 8F\gamma - 9\gamma^2 + 12F\gamma^2 + 3\gamma^3 - 3F\gamma^3}{3(\gamma + 1)(3\gamma^4 - 26\gamma^2 + 52)} \quad (38) \]

The profit expressions for each firm implied by these quantities are provided in the appendix.
3.4 Equilibrium Analysis

As noted above, the strategy set for both firms in the first stage of the game is \( s_i = \{N = \text{no entry}, D = \text{entry with decentralized control}, C = \text{entry with centralized control}\} \). A decision to enter the foreign market by one of the firms (given that the competitor does not have foreign operations) must yield an increase versus the profits earned by the firm when neither firm enters the foreign market (denoted \( \{N, N\} \)). Accordingly, we take the \( \{N, N\} \) equilibrium as our benchmark case and analyze when the firms have a profitable incentive to deviate from this outcome.

For ease of exposition and without loss of generality, we frame the following discussion of best responses in terms of the focal firm, Firm 1. We then derive and discuss the implications of the resulting equilibria.

3.4.1 Equilibrium when only one firm can enter the foreign market

If Firm 1 chooses decentralized foreign entry, the domestic subsidiary chooses \( q_1 \) to maximize \( \pi_1 \), the foreign subsidiary chooses \( q_{F1} \) to maximize \( \pi_{F1} \), and Firm 2 chooses only \( q_2 \) to maximize \( \pi_2 \) (Firm 2 is constrained to operate only in the domestic market in this scenario). With decentralized entry, Firm 1’s profit expression is provided in (24). If Firm 1 chooses centralized foreign entry, the domestic head office chooses \( q_1 \) and \( q_{F1} \) to maximize the sum of \( \pi_1 \) and \( \pi_{F1} \), and Firm 2 chooses only \( q_2 \) to maximize \( \pi_2 \) (because it does not enter the foreign market). With centralized entry, Firm 1’s profit expression is provided in (31). Of course, if Firm 1 chooses not to enter it earns \( \frac{1}{\gamma + 2} \).

Because Firm 1 can earn \( \frac{1}{\gamma + 2} \) without entry, entry only occurs when Firm 1 increases its profits from this level, i.e. when \( F > f_1(\gamma) \) if Firm 1 chooses a decentralized structure or \( F > g_1(\gamma) \) when its structure is centralized.

We define the cutoff value \( B_N \) as the boundary separating the regions where Firm 1’s decentralized profit is higher than its centralized profit and Firm 2 does not enter the foreign market. The results when only one firm has the capability to enter the foreign market are summarized in Proposition 1. To clarify the exposition, it is useful to define \( \gamma_N = \frac{1}{2} \sqrt{2\sqrt{3} - 3} \approx .79623 \). Moreover, the boundary \( B_N \) is comprised of two functions: \( F_N \) and \( \gamma_N \) which we define in the Appendix. The functions \( F_N \) and \( \gamma_N \) are defined over the interval \( [\gamma_N, 1) \), so the equilibrium at \( \gamma = 1 \) is identified separately.

**Proposition 1** When only one firm can enter the market, Firm 1’s best decision is:

1. Non-entry when \( F < g_1(\gamma) \) and \( F < f_1(\gamma) \).

18
2. Centralized entry

(a) when $\gamma < \gamma_N^*$ and $F > g_1(\gamma)$

(b) when $\gamma \in [\gamma_N^*, 1)$ and \textbf{either} $F > \overline{F}_N$ \textbf{or} both $F < \underline{F}_N$ and $F > g_1(\gamma)$.

3. Decentralized entry

(a) when $\gamma \in [\gamma_N^*, 1)$ and $F \in [\underline{F}_N, \overline{F}_N]$ and $F > f_1(\gamma)$

(b) when $\gamma = 1$ and $F > f_1(\gamma)$.

Figure 4 illustrates Firm 1’s best response function when only it has the capability to enter the foreign market. In the region to the right of $B_N$ where $F$ is feasible (i.e. the gray market exists and foreign entry leads to higher profit for Firm 1), decentralized entry is more profitable than centralized entry. In contrast, to the left of $B_N$, centralized entry is more profitable. In other words, when the competing products are highly substitutable and it is in Firm 1’s interest to enter the foreign market, the optimal decision is to enter with a decentralized organization.

This result is surprising given that under decentralization, neither the domestic nor the foreign subsidiary account for the profit implications of their decisions on the subsidiary that operates in the other country. Indeed, the failure to incorporate demand interdependencies leads to both subsidiaries choosing higher quantities than in a centralized setting. In particular, under decentralization, Firm 1’s domestic arm does not account for its quantity choice having a negative impact on
gray market demand (and this indirectly affects the foreign subsidiary’s profitability). Moreover, the aggressive stance of the two subsidiaries under decentralization reduces the size of the total domestic profit pie for the three domestic competitors: Firm 1, Firm 2 and the gray marketer.

The counterpoint to the effect that Firm 1’s decentralization has on the total profit pie is the effect that Firm 1’s decentralization has on Firm 2’s production decision. Because decentralization leads to aggressive production decisions by both of Firm 1’s subsidiaries, decentralization weakens Firm 2: the more substitutable the products are, the more pronounced the effect. The increased level of competition in the domestic market causes Firm 2 to choose a lower production quantity. Recall that under Cournot competition, the decisions of competing firms are strategic substitutes (Tirole 1990). The reduced quantity chosen by Firm 2 leads to higher profit for Firm 1. In other words, foreign entry with a decentralized structure is associated with a tension between two effects:

1. A reduction in the overall profits available for firms in the domestic market, and
2. Firm 1 capturing a larger proportion of the profits that are available.

When $\gamma$ is sufficiently high (i.e., the products are highly substitutable), the increase in the proportion of total profits that accrue to Firm 1 under decentralization more than offsets the reduced size of the pie and leads to higher profits for Firm 1 – both domestically and globally.

Figure 5 shows the impact of the equilibrium organizational structure on the profitability of each subsidiary. To the left of the diagonal bold line, the domestic subsidiary profit is higher

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Figure 5: Comparison of each subsidiary’s profits under decentralized (D) versus centralized (C) structure.
under centralization; above the dashed bold line, the foreign subsidiary profit is higher under centralization. Note that when products are not highly substitutable, both subsidiaries have higher profit under centralization for a large portion of the feasible region. Moving to the right, as products become moderately substitutable, the domestic subsidiary’s profits can be lower under centralization, but the higher foreign profits given centralization are still sufficient to offset lower domestic profits, so that the firm is still better off with the centralization option.

However, to the right of $B_N$, the collective profit shifts to favor decentralization. In the region below the dashed line, both subsidiaries generate higher profit under decentralization. Above the dashed line to the right of $B_N$, the improved domestic subsidiary’s profit under decentralization more than offsets the lower profitability of the foreign subsidiary. In sum, the primary objective of Firm 1 to the left of of $B_N$ is to manage the externality between the foreign market and the domestic market as effectively as possible. In fact, as one moves to the left in Figure 5, Firm 1’s most serious competitor is the gray marketer and not Firm 2. However, to the right of $B_N$, Firm 1’s most serious competitor is Firm 2 and decentralization allows the management of Firm 1 to unleash two aggressive competitors that weaken Firm 2. As a result, Firm 1’s total profit is higher under centralization to the left of $B_N$ and higher under decentralization to the right of $B_N$.

Finally, we note that the dominance of decentralization when products are highly substitutable is not explained by higher gray market volume under centralization. As depicted in Figure 6, with decentralization, gray market quantities can be higher or lower than the quantities observed under centralized control. The dashed line separates the region where gray market volume is higher with centralization compared to decentralization. Above the dashed line, centralization leads to higher gray market volume than decentralization; below the dashed line, the opposite is true.

This observation highlights an important aspect of managing gray markets. For firms, a key benefit of foreign borders (not to mention the geographic distance that often separates countries) is the opportunity it provides to charge different groups of customers different prices. This is critical in the conditions we examine where the "willingness to pay" for two groups of customers is significantly different. Cursory analysis would suggest that the most straightforward approach to retain profits in these conditions is to minimize or eliminate leakage between the groups of customers. In fact, the legal actions we cite in the introduction reflect this perspective on gray markets. Our analysis shows that unless a firm can prohibit gray marketing through legal action

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18 This observation also illustrates that our results are not driven strictly by the inclusion of the gray marketer as an additional player in the domestic game. If they were, we might expect the decentralization decision to lead to strictly higher gray market volume.
Figure 6: Comparison of gray market quantity under decentralization versus centralization

(which is difficult outside of categories like pharmaceuticals), minimizing gray market volume is not the correct criterion. Sometimes the optimal strategy for a firm is to allow a larger gray market volume. This is precisely the case in Figure 6 above the dashed line to the left of $B_N$ and below the dashed line to the right of $B_N$.

The key factor in determining a firm’s optimal organizational structure is the competitiveness of the gray market firm relative to the original domestic competitor. Specifically:

1. When a firm has a significant degree of monopoly power in the domestic market ($\gamma$ is low), the most important criterion is to manage the gray market such that the externality is internalized. This is achieved with a centralized structure.

2. In contrast, when a firm does not enjoy monopoly power ($\gamma$ is high), the most important criterion is to use the gray market as a weapon to weaken the domestic competitor. This is achieved with a decentralized structure.

We now consider a situation where both domestic firms have the ability to enter the foreign market. Similar to this section, we assume that diverted product from a foreign subsidiary is a perfect substitute for the domestic version of that firm’s product.

3.4.2 Equilibrium when both firms can enter the foreign market

When Firm 2 also chooses whether to enter the foreign market and the organizational structure to adopt in the event of entry, the analysis and resulting intuition strongly parallel the decision of Firm
1 in Section 3.4.1 (when only Firm 1 has the ability to enter the foreign market). In particular, if Firm 2 chooses not to enter the foreign market, Firm 1’s optimal entry strategy is depicted in Figure 4. If however, Firm 2 chooses to enter the foreign market and it adopts a decentralized structure, Firm 1’s optimal response in terms of entry and structure is shown in Figure A1 in the appendix. If Firm 2 chooses to enter the foreign market and it adopts a centralized structure, Firm 1’s optimal response is shown in Figure A2 in the appendix.

To clarify the analysis, we define $B_{S_2}$ as the boundary in $(\gamma, F)$ space that delineates the optimal strategy for Firm 1 as a function of $s_2$, Firm 2’s chosen strategy for the foreign market, where $s_2 \in \{N, D \text{ or } C\}$. Not surprisingly, the mathematical definition of $B_{S_2}$ depends on whether Firm 2 centralizes, decentralizes, or does not enter the market. For the three cases, we provide the mathematical definitions of $B_{S_2}$ in the Appendix ($B_N$ is defined in Section 3.4.1. The definitions of $B_D$ and $B_C$ are provided in the proof of Lemma 1). The boundary definitions are indeed different but in all three cases: a) $B_{S_2}$ is a border which divides the parameter space into low and high levels of $\gamma$ and b) $B_{S_2}$ is a threshold to the left of which Firm 1’s optimal organizational structure is centralized, and to the right of which Firm 1’s optimal structure is decentralized (given that the feasibility constraints are satisfied). Similar to the analysis in Section 3.4.1, for each of the three cases, we define a critical value $\gamma^*_{S_2}$ and the two functions: $F_{S_2}$ and $F_{S_2}$ that form the basis for the boundaries (these are defined in the Appendix). We also identify the equilibrium at the endpoint ($\gamma = 1$) separately.\footnote{For the case of C, the functions $F_{S_2}$ and $F_{S_2}$ are defined at $\gamma = 1$ but not for the cases of $N$ and $D$. Accordingly, to simplify exposition of the general case, we identify Firm 1’s optimal strategy at $\gamma = 1$ separately.}

The lemma is written for the general case of $B_{S_2}$ and as in the single firm entry game, the best response is not related to the level of gray market volume.

**Lemma 1** Given a strategy $s_2$ by Firm 2, Firm 1’s optimal strategy is

1. Non-entry when $F < g_1(\gamma)$ and $F < f_1(\gamma)$.

2. Centralized entry

   (a) when $\gamma < \gamma^*_{S_2}$ and $F > g_1(\gamma)$

   (b) when $\gamma \in [\gamma^*_{S_2}, 1)$ and either $F > F_{S_2}$ or both $F < F_{S_2}$ and $F > g_1(\gamma)$.

3. Decentralized entry

   (a) when $\gamma \in [\gamma^*_{S_2}, 1)$ and $F \in [F_{S_2}, F_{S_2}]$ and $F > \min\{g_1(\gamma|N), f_1(\gamma|N)\}$
(b) when $\gamma = 1$ and $F > f_1(\gamma)$.

Although the best responses and resulting intuition parallel the single firm entry situation, there is an important difference between the one firm and two firm entry contexts. In particular, Firm 2 faces a symmetric situation to Firm 1. The objective of the analysis (given a specific set of parameters) is to find a strategy set for the two firms in the first stage of the game where neither firm has an incentive to deviate. As explained earlier, Firm $i$’s strategy is denoted $s_i \in \{N, D \text{ or } C\}$. We define equilibrium as a strategy pair $\{s_1, s_2\}$ where both strategies are best responses to the strategy of the competitor. In certain regions of the parameter space, we find that multiple equilibria are possible. In particular, for an intermediate range of $\gamma$, we find two pure strategy equilibria $\{D, D\}$ and $\{C, C\}$ and the mixed equilibria that obtains when there exist two pure strategy equilibria. For simplicity, when we find multiple equilibria in a given parameter region, we highlight the Pareto optimal equilibrium for that region (details of the other equilibria are provided in the appendix). This is especially pertinent given the starting point of our analysis (i.e. neither firm is operational in the foreign market). Said differently, we assume that firms will not enter the foreign market if $\{N, N\}$ is one of several equilibria, because $\{N, N\}$ is Pareto superior to the other equilibria when it is a fixed point in the first stage.

The boundaries are shown in Figure 7. Beginning with the benchmark case, if the foreign market is too small ($F < \min[g_1(\gamma|N), f_1(\gamma|N)]$), neither firm enters the foreign market. If the foreign market is sufficiently large, and the products are not sufficiently substitutable i.e., $(\gamma, F)$ lies to the left of $B_C$, both firms choose centralized entry.20 Finally, if foreign entry is the equilibrium and the products are sufficiently substitutable i.e., $(\gamma, F)$ lies to the right of $B_C$ then both firms choose decentralized entry. The equilibrium regions, denoted I-IV, are shown in Figure 7 and are summarized as follows.

- **Region I** = $F > g_1(\gamma|N)$ and $(\gamma, F)$ lies to the left of $B_D$.
- **Region II** = $F > g_1(\gamma|N)$ and $(\gamma, F)$ lies to the right of $B_D$ and to the left of $B_C$.
- **Region III** = $F > \min[g_1(\gamma|N), f_1(\gamma|N)]$ and $(\gamma, F)$ lies to the right of $B_C$.
- **Region IV** = $F < g_1(\gamma|N)$ and $F < f_1(\gamma|N)$.

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20 When the products are moderately substitutable i.e., $(\gamma, F)$ lies in Region II of Figure 7, there are multiple equilibria: $\{C, C\}$, $\{D, D\}$ and a mixed strategy equilibrium. We highlight the $\{C, C\}$ equilibrium in this region because it Pareto dominates both $\{D, D\}$ and the mixed strategy equilibrium.
We formalize the result of the game in which both firms can enter the foreign market in Proposition 2.

**Proposition 2** Provided $F$ is feasible:

1. In Region I, both firms choose centralized entry.
2. In Region II, both firms choose centralized entry.
3. In Region III, both firms choose decentralized entry.
4. In Region IV, both firms choose not to enter.

In Region III where $\{D,D\}$ is the unique equilibrium, the firms would realize higher profits were they to both enter with centralized structures. However, in this region the best response to centralized entry by the competitor is decentralized entry and the best response to decentralized entry is also decentralized entry. Thus, Region III constitutes a region characterized by Prisoners’ Dilemma type payoffs. Both firms are worse off because the option of foreign entry with a decentralized structure is available.

We also note that for values of $F$ just above the Region IV boundary $f_1(\gamma|N)$ and $g_1(\gamma|N)$, there exist areas for which a different version of the Prisoner’s Dilemma occurs (i.e., profit is higher for non-entry by both firms, but the equilibrium is either $\{C,C\}$ or $\{D,D\}$). In this region, once a rival firm enters the foreign market, the focal firm suffers substantial cannibalization of its domestic
sales. As a result, its optimal response is to mimic the competitor and enter the foreign market. However, once both firms have entered the foreign market, the firms realize lower profits compared to the profits that would have been realized had neither firm entered the foreign market.

Corollary 1 summarizes the key result of this section. In particular, when the products of Firm 1 and 2 are sufficiently close substitutes and entry is optimal, both firms have a dominant strategy: enter the foreign market and adopt a decentralized organizational structure.

**Corollary 1** When the products of Firm 1 and Firm 2 are sufficiently close substitutes, the unique equilibrium in the feasible range is decentralized entry by both firms.

The primary implication of Corollary 1 is that the equilibrium organizational structure of decentralization does not depend on only one firm having the capability to enter the foreign market (the key finding of Section 3.4.1). Having a decentralized organization is the equilibrium structure when the competing firms (in the domestic market) have products that are highly substitutable. This finding is independent of how many firms enter the foreign market. Naturally, in this model, the choice of organizational structure is irrelevant when there is no gray market: the production decisions of both centralized and decentralized managers are identical when demand in the two countries is independent. However, when there are gray markets, it is interesting to find that decentralization can dominate centralization as a structure for foreign expansion independent of the competitor’s foreign expansion plans. In summary, the analysis shows that the incentive to "internalize" the gray market externality is mitigated by the pre-existing level of competition in the domestic market.

### 3.5 Alternative models of competition

In this section, we assess the impact of alternate forms of competition as a robustness check for the results of our main model.

#### 3.5.1 Gray Market as Competitive Fringe

We first model the gray marketer as a price-taking competitive fringe player. As a fringe player, the gray marketer does not face a downward-sloping demand curve in the domestic market but instead, faces a cost disadvantage that is exacerbated as gray market volume increases.\(^{21}\) Thus, the

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\(^{21}\)The convex cost assumption is an essential element of the classic competitive fringe model. In our setting, convex costs may reflect that an overly active gray marketer attracts unwanted attention from manufacturers, who attempt to shut it down via blacklists, lawsuits or other costly consequences. However, it is possible that gray marketers actually have significant economies of scale in shipping, acquisition and distribution costs.
gray marketer’s profit per unit in this formulation equals the price differential observed between the domestic and foreign markets, minus a supply cost that increases in quantity. The objective functions for the gray marketer, in the one-entry and two-entry case respectively, are replaced with (39) and (40) below:

\[
\pi_G = \left( p_1 - p_{F1} - \frac{1}{2}q_G \right) q_G 
\]

(39)

\[
\pi_G = \left( p_1 - p_{F1} - \frac{1}{2}q_{G1} \right) q_{G1} + \left( p_2 - p_{F2} - \frac{1}{2}q_{G2} \right) q_{G2} 
\]

(40)

The competitive fringe model assumes that the gray marketer fills a profit-maximizing portion of the domestic market demand at the prevailing market price, which is unaffected by gray market sales (because the gray marketer has no market power). Firms 1 and 2 engage in Cournot quantity competition for the residual domestic demand, i.e. the remaining domestic demand other than the portion satisfied by the gray marketer. Note that the foreign market’s consumer demand is not affected by the fringe formulation. Finally, we observe that the remainder of the game, including the sequence of decisions and the objective functions of Firms 1 and 2 remain the same as in the main model.

After choosing organizational structure, both firms choose quantities (local and foreign, as applicable) and the residual market demand determines the resulting price. As in the main model, foreign consumers and the gray marketer pay the same price for product purchased in the foreign market. Because the gray marketer is a price taker by assumption, the gray marketer receives the domestic market price for each firm’s product in the domestic market.

When one firm enters the foreign market, we solve the gray marketer’s problem in (39) with respect to \( q_G \) which, in turn, generates a reaction function for \( q_G \). This reaction function is then substituted into the objective functions for Firm 1’s domestic subsidiary, Firm 1’s foreign subsidiary and Firm 2. These functions are optimized with respect to \( q_1, q_2 \) and \( q_F \), respectively, to create a system of three equations in three unknowns.

When two firms enter the foreign market, the gray marketer’s problem is solved by optimizing profits in (40) with respect to \( q_{G1} \) and \( q_{G2} \). This optimization generates reaction functions for \( q_{G1} \) and \( q_{G2} \). These reaction functions are then substituted into the objective functions of Firms 1 and 2, which are then optimized with respect to \( q_1, q_{F1}, q_2 \) and \( q_{F2} \) respectively to create a system of four equations in four unknowns.

This competitive fringe formulation generates optimal quantities and prices that are identical to the main model. As a result, the optimal profits for Firms 1 and 2 are identical to the corresponding
firm-entry and organization structure settings in the main model. Thus, the competitive fringe formulation produces identical optimal entry and organizational structure choices compared to the main model for both the one-firm and two-firm entry cases.

3.5.2 Price Competition with Capacity Constraints

Our final formulation is a model of Bertrand price competition with capacity constraints where one firm enters the foreign market. In this game, each firm first chooses organizational structure, which determines the decision maker for the capacity and pricing decisions (i.e., the local subsidiary or the head office). Then, each firm chooses production capacities in the domestic and foreign markets. Next, the firms choose prices $p_1$, $p_2$ and $p_F$ in the domestic and foreign markets, and market demand determines the quantity sold at that price. Finally, the gray marketer is a competitive fringe player (and thus a price taker) that maximizes the objective function described in (39) with respect to $q_G$.

This game is solved by backwards induction. Optimizing the gray marketer’s profit function in (39) generates its reaction function $q_G$, which is, in turn, a function of the prices $p_1$ and $p_F$ chosen by the manufacturers in the previous stage. We then substitute this reaction function into the demand functions of Firm 1 and Firm 2, which are identical to the demand functions of the main Cournot model. To ensure that the Bertrand market demand is comparable to the main Cournot model, we invert the demand functions in equations (1) and (2) to obtain the equivalent domestic and foreign demand for each firm as follows:

$$q_1 = \frac{1 - 2p_1 - \gamma(1 - \gamma p_1 - p_2)}{1 - \gamma^2} + p_F \quad (41)$$

$$q_2 = \frac{1 - p_2 - \gamma(1 - p_1)}{1 - \gamma^2} \quad (42)$$

$$q_F = F - p_F \quad (43)$$

As in the Cournot model, gray market demand, $q_G$, is determined endogenously and is incremental to local demand from foreign customers, $q_F$. However, in contrast to the Cournot game, the objective functions of Firms 1 and 2 are optimized with respect to prices $p_1$, $p_2$ and $p_F$ in the

---

22 See Appendix for an outline of the proof. One difference in the competitive fringe setting, however, is that the gray market firm has strictly lower profits than in the main model setting.

23 We note that, unlike the main model setting which allows for a Stackelberg follower, it is difficult to justify the gray marketer as a sequential price setter. The reason is that the manufacturers’ prices are not best responses to the gray marketer’s price when the gray marketer chooses price last. Accordingly, the gray market firm is modelled as a price-taking competitive fringe player.
pricing game. Finally, the optimal prices $p_1$, $p_2$ and $p_F$ influence the production capacities chosen by the manufacturers in the first stage.

In equilibrium, we find that firms install precisely the capacity that is demanded given their chosen prices. On the one hand, there is no incentive to install excess capacity that would be idle; on the other hand, if firms install less capacity than the quantity demanded for a given price point, the firms would leave unsatisfied demand and would be better off choosing a higher price. To solve the game technically with capacity constraints, the pricing reaction functions for the end game are determined based on the capacities selected in the first stage. This leads to a reaction function for each manufacturer with a discontinuity at the point where the optimal price ($p^*$) leads to demand exactly equal to the capacity selected in the first stage. At prices less than $p^*$, each firm has a reaction function that is "flatter" reflecting a reduced incentive to cut price when a firm operates at capacity. In fact, at prices less than $p^*$, the optimal reaction is for the firm to set price such that the quantity demanded equals capacity (precisely). This structure effectively allows a firm to "choose" a point on the competitor’s reaction function that optimizes its profit (by restricting capacity, the firm moves the discontinuity away from the origin). It is straightforward to demonstrate that both firms have an incentive to restrict capacity when capacities are set at the quantities that would be chosen in a game without capacity restrictions.

In an untabulated technical appendix we show that, in the one-entry case, the Bertrand model with capacity constraints generates identical optimal quantities and prices to the main model. In addition, the profit functions and optimal profits for Firms 1 and 2 in the Bertrand model are the same as in the main model, independent of either firm’s entry or organizational structure decision. As a result, the Bertrand model with capacity constraints leads to optimal entry and organizational structure results that are identical to the main model.

3.6 Total Welfare

In an untabulated technical appendix, we illustrate that total welfare (i.e., aggregate consumer and producer surplus across both the domestic and foreign economies) is strictly higher when firms enter the foreign market with a decentralized, rather than a centralized, organizational structure. This result makes sense because, although gray market volume may be higher or lower with decentralized entry, total volume (i.e., aggregate quantity produced across all firms) is strictly higher.

24 The two-firm-entry case parallels the one-entry case and is also similar to the analysis proposed by Kreps and Scheinkman (1983). As in Section 3.5.1, a competitive fringe gray market firm (this time in the Bertrand setting with capacity constraints) has lower profits compared to the main model.
with decentralized entry. A higher global volume leads to greater global competition, lower market prices, greater global consumer surplus, and higher total global welfare. Moreover, in the situation where only one firm can enter the foreign market and the competing products are highly substitutable, decentralized entry is not only profit-maximizing for the focal firm, but also increases total welfare.

4 Conclusion

In this section, we review the key insights of our analysis and highlight several limitations that are useful to interpret the findings. We then provide a closing summary.

4.1 Key Insights

The fundamental insight provided by our analysis is that multinational firms can use organizational structure - in particular, the decision rights over who sets production levels - to manage competition in the presence of gray markets. In a centralized organization, the head office controls production choices, whereas in a decentralized organization, the individual subsidiaries are granted autonomy over production to maximize local performance.

When gray markets are active, we document a tension between maximizing the global industry profit pie and maximizing a firm’s share of global industry profits. Maximizing the global industry profit pie occurs with centralized control due to "internalization" of the externality created by gray markets. Maximizing a firm’s share of the global profit pie occurs with decentralized control due to the more aggressive decisions of locally motivated decision makers. The optimal organizational structure is driven by how fiercely firms compete, i.e. the level of differentiation between competing products. In our model, differentiation is represented by the degree of substitutability between the two domestic products. When competing products are highly substitutable (the level of differentiation between products is low), the primary need of each firm is to capture as much of the market as possible. This is achieved with decentralized control. Decentralized control leads to more aggressive decisions by local managers, which in turn leads to defensive responses from the competitor. This outcome is advantageous when the level of substitutability is high.

Importantly, this finding is not the result of one structure leading to less activity in the gray market. Even when decentralization is the optimal structure, the resulting gray market volume can be higher or lower compared to the volume observed under centralization. The key concern that affects the optimal structure is to control the factor which has the greatest negative effect on
firm profits. When the level of competition between domestic products is intense, the factor which has the strongest negative effect on firm profits is the competitor. Here, decentralized structure is superior to weaken the competitor. In contrast, when the level of competition between products is low, the factor which has the most negative effect on firm profits is the gray market. The impact of this factor is best mitigated with centralized control which internalizes the negative effect of the gray market.

4.2 Limitations

As with all analytical models, these results are subject to limitations. For tractability, we restrict the analysis to an industry where all subsidiaries make their production decisions simultaneously. In some cases, this may not accurately reflect how production decisions are made in a multinational firm, especially when the fraction of foreign production that leaks back to the domestic market can be forecast with a high degree of accuracy. We have conducted preliminary analysis of a situation where the fraction of leakage is known *ex ante*. While there are subtle differences in the findings, we continue to find that when the competing products in the domestic market are highly substitutable, decentralized control is beneficial for firms that enter the low-priced foreign market.

We also abstract away from individual players that may be active in the distribution channel. In particular, resellers and distributors may participate in gray market activity and manufacturers may employ strategies to curtail gray market activity outside of quantity and pricing decisions (e.g., lobbying regulators, conducting investigations, penalties for breaching contracts by selling outside authorized channels, etc.). These issues are not reflected in our analysis.

Finally, our single-period model does not reflect the intertemporal tradeoffs that firms contemplating global expansion consider in their planning. For example, our model is static and does not reflect the potential growth that may occur in a foreign market or the value that gaining an early foothold in such a market may deliver (despite the immediate problem it creates for the firm in terms of gray market goods). We leave further analysis arising from these limitations to the future.

4.3 Summary

As the global firm is the ultimate originator of all gray market goods, the gray market problem appears to have an obvious solution. Why not centralize the production decision of the foreign subsidiary to minimize the negative impact of the gray market? A centralized decision maker would account for the gray market in making production decisions. In contrast to this thinking,
our analysis demonstrates that there can be a structural advantage to decentralizing production decisions in the presence of endogenous leakage across markets. This result obtains when competing products are highly substitutable. Accordingly, this study identifies new insight into the empirical regularity that global firms often empower their overseas subsidiaries to operate independently and set production levels even when the threat of foreign production being diverted back to the domestic market is high.

Three managerial implications follow from our analysis. First, when a firm plans to expand internationally, the degree to which competitive products are close substitutes should play a role in the organizational structure decision, in addition to factors such as local knowledge, the need to motivate local management and regulatory restrictions. Second, while coordination between competitors can violate antitrust law within a country, there is little supra-national legal restriction regarding a firm’s strategic choice of organizational structure during entry into a foreign market. Finally, managers should not necessarily be instructed to minimize gray market volume as sometimes profits are optimized with higher gray market volume.

We also demonstrate that, although global profits can be higher in a world where all firms centralize their production decisions (or in some cases where no firms enter the foreign market), the strategic advantage of decentralization may cause firms to deviate from the centralization (or non-entry) path. In a market where competing products are highly substitutable, decentralization is appealing precisely because it provides a firm the opportunity to steal business from a competitor that employs centralized control or only has domestic operations. However, when the competitor has the ability to neutralize the business-stealing advantage of decentralization (by decentralizing itself), the benefit of decentralizing disappears and firms are strictly worse off.
References


Appendix

Sketch of Solution in the Decentralized Case where Firm 1 enters the Foreign Market

The gray marketers problem is to optimize $\pi_G = (p_1 - p_F) q_G$ with respect to the choice of $q_G$. This implies that the gray marketer’s optimization is:

$$\frac{\partial \pi_G}{\partial q_G} = \frac{\partial}{\partial q_G} \left( q_G - F q_G - q_1 q_G + q_F q_G - \gamma q_2 q_G - q_G^2 \right) = 0$$

$$\Rightarrow q_G = \frac{1}{2} (q_F - q_1 - \gamma q_2 + 1 - F)$$

(i)

Substituting in $q_G$ and differentiating the objective functions for Firm 1 (the domestic and foreign profit functions) and Firm 2, we obtain the following first order conditions.

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1}{2} F - q_1 - \frac{1}{2} q_F - \frac{1}{2} \gamma q_2 + \frac{1}{2} = 0$$

$$\frac{\partial \pi_2}{\partial q_2} = -\frac{1}{2} \gamma + \frac{1}{2} F \gamma - 2 q_2 - \frac{1}{2} \gamma q_1 - \frac{1}{2} \gamma q_F + \gamma^2 q_2 + 1 = 0$$

$$\frac{\partial \pi_G}{\partial q_F} = 2 F + \frac{1}{2} q_1 - 3 q_F + \frac{1}{2} \gamma q_2 - \frac{1}{2} = 0$$

(ii)

Solving these expressions for $q_1$, $q_2$, and $q_F$, leads to the unique solution provided in the main text. In order for Firm 1 to enter the market, $\Pi_1 > \frac{1}{(\gamma+2)^2}$ is a necessary condition. This implies that $F > f_1(\gamma)$ is a necessary condition where

$$f_1(\gamma) = \frac{-208 \gamma + 496 \gamma^2 + 300 \gamma^3 - 118 \gamma^4 - 108 \gamma^5 - 18 \gamma^6 - 416}{151 424 \gamma - 375 648 \gamma^2 - 381 056 \gamma^3 + 347 088 \gamma^4 + 369 968 \gamma^5 + 2 \gamma^6 - 169 664 \gamma^7 + 18 501 \gamma^8 + 35 136 \gamma^9 + 1392 \gamma^{10} - 2304 \gamma^{11} - 192 \gamma^{12}}$$

$$+ \frac{832 \gamma - 848 \gamma^2 - 1056 \gamma^3 + 72 \gamma^4 + 336 \gamma^5 + 84 \gamma^6 + 832}{151 424}$$

(iii)

In addition, for the gray market to exist, $p_1 > p_F$ is a necessary condition. This implies that:

$$F < \frac{10 \gamma + 5 \gamma^2 - 20}{14 \gamma^2 - 24} = m_1(\gamma)$$

Sketch of Solution in the Decentralized Case where both firms enter the Foreign Market

The gray marketers problem is to optimize $\pi_G = (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2}$ with respect to the choice of $q_{G1}$ and $q_{G2}$. This leads to the following first order conditions for the gray marketer:

$$\frac{\partial \pi_G}{\partial q_{G1}} = -F - 2 q_{G1} + q_{F1} - q_1 - 2 \gamma q_{G2} + \gamma q_{F2} - \gamma q_2 + 1 = 0$$

(iv)

$$\frac{\partial \pi_G}{\partial q_{G2}} = -F - 2 q_{G2} + q_{F2} - q_2 - 2 \gamma q_{G1} + \gamma q_{F1} - \gamma q_1 + 1 = 0$$

(v)

This leads to the following reaction functions for the gray marketer:
\[ q_{G1} = \frac{1}{2\gamma + 2} (q F_1 - F - q_1 + \gamma q F_1 - \gamma q_1 + 1) \]
\[ q_{G2} = \frac{1}{2\gamma + 2} (q F_2 - F - q_2 + \gamma q F_2 - \gamma q_2 + 1) \] (vi)

Substituting and differentiating the objective functions for Firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first order conditions.

\[ \frac{\partial \pi_1}{\partial q_1} = -\frac{3}{2} q_1 + \frac{1}{2\gamma + 2} (F - q F_1 + q_1 - \gamma q F_1 + \gamma q_1 - 1) \]
\[ + \frac{1}{2\gamma + 2} (-\gamma + F\gamma - \gamma q F_2 - \gamma q_2 - \gamma^2 q F_2 - \gamma^2 q_2) + 1 = 0 \] (vii)

\[ \frac{\partial \pi_2}{\partial q_2} = -\frac{3}{2} q_2 + \frac{1}{2\gamma + 2} (F - q F_2 + q_2 - \gamma q F_2 + \gamma q_2 - 1) \]
\[ + \frac{1}{2\gamma + 2} (-\gamma + F\gamma - \gamma q F_1 - \gamma q_1 - \gamma^2 q F_1 - \gamma^2 q_1) + 1 = 0 \] (viii)

\[ \frac{\partial \pi_{F1}}{\partial q F_1} = \frac{3}{2} F - \frac{5}{2} q F_1 - \frac{3}{2} \gamma q F_2 + \frac{1}{2\gamma + 2} (F - q F_1 + q_1 - \gamma q F_1 + \gamma q_1 - 1) = 0 \] (ix)

\[ \frac{\partial \pi_{F2}}{\partial q F_2} = \frac{3}{2} F - \frac{5}{2} q F_2 - \frac{3}{2} \gamma q F_1 + \frac{1}{2\gamma + 2} (F - q F_2 + q_2 - \gamma q F_2 + \gamma q_2 - 1) = 0 \] (x)

Solving these expressions for \( q_1, q_2, q F_1 \) and \( q F_2 \), leads to the unique solution provided in the main text.

**Sketch of Solution in the Centralized Case where Firm 1 enters the Foreign Market**

Substituting and differentiating the objective functions for Firms 1 and 2, we obtain the following first order conditions.

\[ \frac{\partial \Pi_1}{\partial q_1} = -q_1 - \frac{1}{2}\gamma q_2 + \frac{1}{2} = 0 \]
\[ \frac{\partial \Pi_2}{\partial q_2} = -\frac{1}{2}\gamma + \frac{1}{2} F\gamma - 2 q_2 - \frac{1}{2}\gamma q_1 - \frac{1}{2}\gamma q F + \gamma^2 q_2 + 1 = 0 \] (xi)

Solving these expressions for \( q_1, q_2, q F_1 \) and \( q F_2 \), leads to the unique solution provided in the main text.

In order for Firm 1 to enter the market, \( \Pi_1 > \frac{1}{(\gamma + 2)^2} \) is a necessary condition. This implies that \( F > g_1(\gamma) \) is a necessary condition where

\[ g_1(\gamma) = \frac{240\gamma^2 - 96\gamma + 144\gamma^3 - 60\gamma^4 - 54\gamma^5 - 9\gamma^6}{2 (19\gamma^4 - 60\gamma^2 + 48) (\gamma + 2)^2} \] (xii)

In addition, for the gray market to exist, \( p_1 > p F \) is a necessary condition. This implies that:

\[ F < \frac{2\gamma^2 + \gamma^4 - 4}{2\gamma^2 - 4} = n_1(\gamma). \]
Sketch of Solution in the Centralized Case where Both Firms enter the Foreign Market

The gray marketer’s problem is unaffected by the organizational structure of the competing firms so the reactions function for the gray marketer in the centralized conditions are identical to equation (vi). Substituting and differentiating the objective functions for Firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first order conditions.

\[
\frac{\partial \Pi_1}{\partial q_1} = -q_1 - \frac{1}{2} \gamma q_2 + \frac{1}{2} = 0 \quad (xiii)
\]

\[
\frac{\partial \Pi_2}{\partial q_2} = -q_2 - \frac{1}{2} \gamma q_1 + \frac{1}{2} = 0 \quad (xiv)
\]

\[
\frac{\partial \Pi_1}{\partial q_{F1}} = -\frac{1}{2} q_1 + \frac{1}{2 \gamma + 2} (4F + 3F\gamma - 6q_{F1} + q_1 - 6\gamma q_{F1} - 3\gamma q_{F2} + \gamma q_1 - 3\gamma^2 q_{F2} - 1) = 0 \quad (xv)
\]

\[
\frac{\partial \Pi_2}{\partial q_{F2}} = -\frac{1}{2} q_2 + \frac{1}{2 \gamma + 2} (4F + 3F\gamma - 6q_{F2} + q_2 - 3\gamma q_{F1} - 6\gamma q_{F2} + \gamma q_2 - 3\gamma^2 q_{F1} - 1) = 0 \quad (xvi)
\]

Solving these expressions for \(q_1, q_2, q_{F1}\) and \(q_{F2}\), leads to the unique solution provided in the main text.

Sketch of Solution to the Asymmetric Case where Both Firms enter the Foreign Market

As before, the gray marketer’s problem is unaffected by the organizational structure of the competing firms so the reactions function for the gray marketer in the centralized conditions are identical to equation (vi). Substituting and differentiating the objective functions for Firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first order conditions.

\[
\frac{\partial \Pi_1}{\partial q_1} = -q_1 - \frac{1}{2} \gamma q_2 + \frac{1}{2} = 0 \quad (xvii)
\]

\[
\frac{\partial \Pi_2}{\partial q_2} = \frac{1}{2} F - \frac{1}{2} q_{F2} - q_2 - \frac{1}{2} \gamma q_{F1} - \frac{1}{2} \gamma q_1 + \frac{1}{2} = 0 \quad (xviii)
\]

\[
\frac{\partial \Pi_1}{\partial q_{F1}} = \frac{4F + 3F\gamma - 6q_{F1} - 6\gamma q_{F1} - 3\gamma q_{F2} - 3\gamma^2 q_{F2} - 1}{2\gamma + 2} = 0 \quad (xix)
\]

\[
\frac{\partial \Pi_{F2}}{\partial q_{F2}} = \frac{4F + 3F\gamma - 6q_{F2} + q_2 - 3\gamma q_{F1} - 6\gamma q_{F2} + \gamma q_2 - 3\gamma^2 q_{F1} - 1}{2\gamma + 2} = 0 \quad (xx)
\]

Solving these expressions for \(q_1, q_2, q_{F1}\) and \(q_{F2}\), leads to the unique solution provided in the main text. These lead to the following profit expressions for each of the three firms. To highlight the different structures associated with each firm, we replace the firms identifiers, 1 and 2, with the subscripts \(cent\) and \(decent\).
When the market characteristics are given in (24), from centralized entry is given in (31), and non-entry profit is:

\[
\pi_G = \frac{12272F\gamma - 5824\gamma - 1404F + 7888F^2 - 2400\gamma^2 + 3396\gamma^3 - 207\gamma^4 - 495\gamma^5 + 108\gamma^6 + 6360\gamma^7 - 6448F^2\gamma - 7776F^2\gamma^3 + 204F\gamma^4 + 1224F\gamma^5 - 252F\gamma^6 - 4176F^2\gamma^2 + 4416F^2\gamma^3 - 756F^2\gamma^4 + 144F^2\gamma^5 + 6304}{36(3\gamma^4 - 26\gamma^2 + 52)^2(\gamma + 1)}
\]

**Proof of Proposition 1**

When only Firm 1 is capable of entering the foreign market, Firm 1’s profit from decentralized entry is given in (24), from centralized entry is given in (31), and non-entry profit is \(\frac{1}{(\gamma + 2)^2}\). We previously showed that \(\Pi^D_1 > \frac{1}{(\gamma + 2)^2}\) if and only if \(F > f_1(\gamma)\) and \(\Pi^C_1 > \frac{1}{(\gamma + 2)^2}\) if and only if \(F > g_1(\gamma)\) (see derivations at (iii) and (xii)). Therefore when both \(F < f_1(\gamma)\) and \(F < g_1(\gamma)\), Firm 1’s best option is non-entry. This establishes claim 1.

Next, we derive the boundary \(B_N\) by calculating \(\Pi^D_1 - \Pi^C_1\), which is quadratic in \(F\). Solving for \(F^*\) yields an expression in \(\gamma\) with the following discriminant.

\[
DS_N = \sqrt{-(2\gamma^4 - 6\gamma^2 + 3)(374\gamma^3 - 904\gamma^2 - 312\gamma + 411\gamma^4 - 112\gamma^5 - 56\gamma^6 + 624)^2}
\]  

The discriminant \(DS_N = 0\) when \(\gamma = \gamma^*_N = \frac{1}{2}\sqrt{2\sqrt{3} - 3 - \sqrt{3}}\). At \(\gamma^*_N\), \(F^* = \frac{3}{2\sqrt{3}}\left(60 + \sqrt{3} - 2\sqrt{1611 - 807\sqrt{3}}\right)\). \(F^*\) has complex roots when \(\gamma < \gamma^*_N\) and \(\gamma = 1\). The upper and lower roots of \(F^*\) are as follows:

\[
F_N = \frac{312\gamma + 2304\gamma^2 - 1074\gamma^3 - 2501\gamma^4 + 982\gamma^5 + 1031\gamma^6 - 270\gamma^7 - 135\gamma^8 + \sqrt{-(2\gamma^4 - 6\gamma^2 + 3)(374\gamma^3 - 904\gamma^2 - 312\gamma + 411\gamma^4 - 112\gamma^5 - 56\gamma^6 + 624)^2}}{2(1260\gamma^2 - 843\gamma^4 + 187\gamma^6 - 624)(\gamma + 1)(\gamma - 1)}
\]  

\[
F_N = \frac{312\gamma + 2304\gamma^2 - 1074\gamma^3 - 2501\gamma^4 + 982\gamma^5 + 1031\gamma^6 - 270\gamma^7 - 135\gamma^8}{2(1260\gamma^2 - 843\gamma^4 + 187\gamma^6 - 624)(\gamma + 1)(\gamma - 1)}
\]

When \(\gamma > \gamma^*_N\), the boundary \(B_N\) is defined by \(F_N\) for \(F > \frac{3}{2\sqrt{3}}\left(60 + \sqrt{3} - 2\sqrt{1611 - 807\sqrt{3}}\right)\) and \(F_N\) for \(F < \frac{3}{2\sqrt{3}}\left(60 + \sqrt{3} - 2\sqrt{1611 - 807\sqrt{3}}\right)\). When the market characteristics \((\gamma, F)\) lie to the right of \(B_N\) and \(F > f_1(\gamma)\) then Firm 1’s profit is higher under decentralization than centralization or no entry. This establishes claim 3. Conversely, when the market characteristics \((\gamma, F)\) lie to the left of \(B_N\) and \(F > g_1(\gamma)\), Firm 1 is better off under centralized entry. This establishes claim 2 and completes the proof of Proposition 1.
Proof of Lemma 1

We consider 3 cases: (a) Firm 2 chooses non-entry, (b) Firm 2 chooses decentralized entry, and (c) Firm 2 chooses centralized entry. For each case, we derive the respective boundary as in Proposition 1 by calculating the cutoff $F$ where $\Pi_1^D - \Pi_1^C$ (given Firm 2’s choice) changes sign. $B_N$ is given in equations xxii and xxiii. The remaining boundaries $B_D$ and $B_C$ are determined as follows:

First, we derive the boundary $B_D$ by calculating $\Pi_1^D - \Pi_1^C$ when Firm 2 has entered the foreign market with decentralized control. Solving this expression for $F$ yields an expression in $\gamma$ with the following discriminant.

$$DS_D = \sqrt{3 \left(-169 + 338\gamma^2 + 338\gamma^4 - 234\gamma^6 + 27\gamma^8 \right)} \left[(52 - 26\gamma^2 + 3\gamma^4)(13 + 13\gamma + 3\gamma^2)\right]^2$$ (xxiv)

The discriminant $DS_D = 0$ when $\gamma = \gamma_D^* = \frac{1}{3} \sqrt{\frac{1}{2} \left(39 - 7\sqrt{39} + \sqrt{78 \left(8 - \sqrt{39}\right)}\right)}$. At $\gamma_D^*$, $F^* = \frac{1}{12} \sqrt{117} - \frac{1}{4}$. $F^*$ has complex roots when $\gamma < \gamma_N^*$ and $\gamma = 1$. The upper and lower roots of $F^*$ are as follows:

$$F_D = \left(\begin{array}{c}
-8788 - 4056\gamma + 43940\gamma^2 + 32448\gamma^3 - 22984\gamma^4 - 22542\gamma^5 \\
+ 1911\gamma^6 + 5148\gamma^7 + 612\gamma^8 - 378\gamma^9 - 81\gamma^{10}
\end{array}\right)$$

$$+ \sqrt{3 \left(-169 + 338\gamma^2 + 338\gamma^4 - 234\gamma^6 + 27\gamma^8 \right)} \left[(52 - 26\gamma^2 + 3\gamma^4)(13 + 13\gamma + 3\gamma^2)\right]^2$$ (xxv)

When $\gamma > \gamma_D^*$, the boundary $B_D$ is defined by $F_D$ for $\gamma > \frac{1}{4} \sqrt{13} - \frac{1}{4}$ and $E_D$ for $\gamma < \frac{1}{4} \sqrt{13} - \frac{1}{4}$. When the market characteristics $(\gamma, F)$ lie to the right of $B_D$ and $F > \text{Min}[g_1(\gamma|N), f_1(\gamma|N)]$ then Firm 1’s profit is higher under decentralization than centralization or no entry. Conversely, when the market characteristics $(\gamma, F)$ lie to the left of $B_D$ and $F > g_1(\gamma)$, Firm 1 is better off under centralized entry.

We next derive the boundary $B_C$ by calculating $\Pi_1^C - \Pi_1^C$ when Firm 2 has entered the foreign market with centralized control. Solving this expression for $F$ yields an expression in $\gamma$ with the following discriminant.

$$DS_C = \sqrt{3 \left(-16 + 16\gamma^2 + 44\gamma^4 - 24\gamma^6 + 3\gamma^8 \right)} \left[(52 - 26\gamma^2 + 3\gamma^4)\right]^2$$ (xxvii)

The discriminant $DS_C = 0$ when $\gamma = \gamma_C^* = \frac{1}{3} \sqrt{3 \left(6 - \sqrt{3} \sqrt{7 - \sqrt{13}}\right)}$. At $\gamma_C^*$, $F^* = \frac{1}{4} \sqrt{13} - \frac{1}{4}$. $F^*$
has complex roots when $\gamma < \gamma_N^*$. The upper and lower roots of $F^*$ are as follows:

$$
F_C = \frac{208 - 80\gamma^2 + 544\gamma^4 - 120\gamma^6 + 9\gamma^8}{8 (-52 + 76\gamma^2 - 28\gamma^4 + 3\gamma^6)}
$$

(xxviii)

$$
F_C = \frac{208 - 80\gamma^2 + 544\gamma^4 - 120\gamma^6 + 9\gamma^8}{8 (-52 + 76\gamma^2 - 28\gamma^4 + 3\gamma^6)}
$$

(xxix)

When $\gamma > \gamma_C^*$, the boundary $B_C$ is defined by $F_C$ for $F > \frac{1}{15}\sqrt{13} - \frac{1}{15}$ and $E_C$ for $F < \frac{1}{15}\sqrt{13} - \frac{1}{15}$.

When the market characteristics $(\gamma, F)$ lie to the right of $B_C$ and $F > Min[g_1(\gamma|N), f_1(\gamma|N)]$ then Firm 1’s profit is higher under decentralization than centralization or no entry. Conversely, when the market characteristics $(\gamma, F)$ lie to the left of $B_C$ and $F > g_1(\gamma)$, Firm 1 is better off under centralized entry.

To summarize:

$$
B_{S2} = \begin{cases} 
B_N & \text{if Firm 2 chooses non-entry} \\
B_D & \text{if Firm 2 chooses decentralized entry} \\
B_C & \text{if Firm 2 chooses centralized entry} 
\end{cases}
$$

When the market characteristics $(\gamma, F)$ lie to the left of $B_{S2}$, then Firm 1’s profit is higher under centralized entry than under decentralized entry. Conversely, when the market characteristics $(\gamma, F)$ lie to the right of $B_{S2}$, then Firm 1’s profit is higher under decentralized entry than under centralized entry. Provided entry is preferred to non-entry, Firm 1’s best choice is determined by the location of the market characteristics in relation to the relevant boundary. This proves Lemma 1.

**Sketch of Proof of Proposition 2**

This proof proceeds as follows. First, we establish the details of Firm 1’s best response function. Second, we solve for all the equilibrium regions and for any region with multiple equilibria, we determine the Pareto optimal equilibrium.

1. When both firms can enter the foreign market, Firm 2 has three possible choices: no entry, decentralized entry, and centralized entry. We modify our notation to denote Firm 2’s choice as follows: "$|N", "$|D", and "$|C". When Firm 2 chooses no entry, Firm 1’s best responses are given in Proposition 1. The decentralized and centralized entry responses are solved in a similar manner, resulting in the following expressions.
We illustrate Firm 1's best response functions for decentralized and centralized entry in Figures A1 and A2 respectively. Figures A1 and A2 respectively also show the respective infeasible regions above the decentralized cutoff $m_1(\gamma|D)$ where the $q_G \leq 0$; however, the corresponding cutoff with centralized entry never binds, i.e., $n_1(\gamma|C) \geq 1$. 

\begin{align*}
  g_1(\gamma|D) &= \frac{\left(-1827904 + 3233984\gamma + 1379040\gamma^2 - 4937504\gamma^3 + 786864\gamma^4 + 2517008\gamma^5 - 903448\gamma^6 - 480792\gamma^7 + 226932\gamma^8 + 32004\gamma^9 - 19134\gamma^{10} - 702\gamma^{11} + 513\gamma^{12}\right)}{2(1827904 - 1968512\gamma - 2547168\gamma^2 + 3266432\gamma^3 - 940992\gamma^4 - 1881152\gamma^5 + 53560\gamma^6 + 439296\gamma^7 - 72132\gamma^8 - 36864\gamma^9 + 8694\gamma^{10} + 27\gamma^{12})} \\
  g_1(\gamma|C) &= \frac{\left(\begin{array}{c}
  -144 + 72\gamma + 192\gamma^2 - 66\gamma^3 \\
  -73\gamma^4 + 15\gamma^5 + 6\gamma^6 \\
  + 3(-12 + 7\gamma)^2(-144 + 144\gamma)
\end{array}\right)}{576 - 696\gamma^2 - 24\gamma^3 + 202\gamma^4 + 12\gamma^5 + 3\gamma^6} \\
  f_1(\gamma|D) &= \frac{\left(\begin{array}{c}
  -7436\gamma + 54418\gamma^2 + 15574\gamma^3 - 24219\gamma^4 \\
  -8640\gamma^5 + 2323\gamma^6 + 1248\gamma^7 + 171\gamma^8
\end{array}\right)}{2\left(35152 + 32448\gamma - 35490\gamma^2 - 40612\gamma^3 + 3003\gamma^4 + 12470\gamma^5 + 3301\gamma^6 + 78\gamma^7 + 9\gamma^8\right)} \\
  f_1(\gamma|C) &= \frac{\left(\begin{array}{c}
  -59904 + 92352\gamma + 47104\gamma^2 - 132384\gamma^3 + 16864\gamma^4 \\
  + 64592\gamma^5 - 23144\gamma^6 - 12504\gamma^7 + 6376\gamma^8 + 104\gamma^9 - 690\gamma^{10} - 54\gamma^{11} + 36\gamma^{12}
\end{array}\right)}{\sqrt{-3(1248 - 624\gamma - 1352\gamma^2 + 676\gamma^3 + 436\gamma^4 - 218\gamma^5 - 42\gamma^6 + 21\gamma^7)^2}((-2688 + 5184\gamma - 224\gamma^2 - 4320\gamma^3 + 1336\gamma^4 + 1232\gamma^5 - 398\gamma^6 - 192\gamma^7 + 67\gamma^8)^2)}}{119808 - 119808\gamma - 170528\gamma^2 + 195072\gamma^3 + 71536\gamma^4 - 112192\gamma^5 - 3512\gamma^6 + 26880\gamma^7 - 3068\gamma^8 - 2352\gamma^9 + 414\gamma^{10} + 9\gamma^{12}}
\end{align*}
Figure A1: $F$ constraints for Firm 1 given decentralized entry by Firm 2: no decentralized entry for $F < f_1(\gamma)$; no centralized entry for $F < g_1(\gamma)$.

Figure A2: $F$ constraints for Firm 1 given centralized entry by Firm 2: No decentralized entry for $F < f_1(\gamma)$; no centralized entry for $F < g_1(\gamma)$.

1. The equilibrium regions determined as follows. Figure A3 depicts these regions; areas with multiple equilibria are shown with a ***.
Figure A3: Details of equilibrium regions when both firms can enter foreign market

(a) Region I versus Region IV. We begin by noting that for any given level of $F$, $\gamma_{BD} < \gamma_{BC} < \gamma_{BN}$ (in plain terms $B_D$ is strictly to the left of $B_C$ and $B_C$ is strictly to the left of $B_N$). To the left of $B_D$, if Firm 2 chooses either decentralized or centralized entry, then Firm 1’s best response is centralized entry or no entry. When will entry be preferred? Since $\gamma_{BC} < \gamma_{BN}$, for all $F > g_1(\gamma|N)$ entry will be uniquely preferred, and $\{C, C\}$ is the unique Nash equilibrium. When the market characteristics $(\gamma, F)$ lie to the left of $B_D$ and $F < g_1(\gamma|C)$, the unique Nash equilibrium is $\{N, N\}$. When the market characteristics $(\gamma, F)$ lie to the left of $B_D$ and $F$ lies between $g_1(\gamma|C)$ and $g_1(\gamma|N)$, there are 3 equilibria, $\{C, C\}$, $\{N, N\}$ and a mixed strategy. In this parameter region, $\Pi_1^{NN} > \Pi_1^{CC}$ so we highlight the Pareto optimal equilibrium $\{N, N\}$ as being the most probable.

(b) Region II versus Region IV. When the market characteristics $(\gamma, F)$ lie to the right of $B_D$ but to the left of $B_C$, there are 3 equilibria. First, consider the cases when the relevant entry condition is met (i.e., $F > g_1(\gamma|N)$ ). If Firm 2 chooses $D$, then because the market characteristics are to the right of $B_D$, Firm 1 will choose $D$ as well, yielding a $\{D, D\}$ equilibrium (provided $F < m_1(\gamma|D)$). If Firm 2 chooses $C$, then because the market characteristics are to the left of $B_C$, Firm 1 will choose $C$ yielding a $\{C, C\}$ equilibrium. As before there is also a mixed equilibrium. In this region, $\Pi_1^{CC}$ is the most profitable of the three equilibria, so we opt for the Pareto optimal equilibrium $\{C, C\}$ here. Next we consider the non-entry regions. When the market characteristics $(\gamma, F)$ lie to the right of $B_D$ but to the left of $B_C$ and $F < g_1(\gamma|C)$, the unique Nash equilibrium is $\{N, N\}$. When the market characteristics $(\gamma, F)$ lie to the right of $B_D$ but to the left of $B_C$ and $F$ is between $g_1(\gamma|C)$ and $g_1(\gamma|N)$, there are multiple equilibria, as well as several mixed strategies. Of all these equilibria, $\Pi_1^{NN}$ is the most profitable,

\footnote{This equilibrium refinement also creates the most conservative possible boundary for our counterintuitive equilibrium, $\{D, D\}$.}
so we highlight the Pareto optimal equilibrium \{N, N\} as being the most probable.

(c) Region III versus Region IV. When the market characteristics \((\gamma, F)\) lie to the right of \(B_C\) and \(F < m_1(\gamma|D)\), if Firm 2 chooses either decentralized or centralized entry, then Firm 1’s best response is decentralized entry or no entry. When will entry be preferred? Since \(f_1(\gamma|D) < \min[f_1(\gamma|N), g_1(\gamma|N)]\), for all \(F > f_1(\gamma|N)\) and \(F > g_1(\gamma|N)\), entry will be uniquely preferred, and \{D, D\} is the unique Nash equilibrium. For \(F < \max[f_1(\gamma|D), d(\gamma)]\), the unique Nash equilibrium is \{N, N\}. For all \(F\) between \(\max[f_1(\gamma|D), d(\gamma)]\) and \(\min[f_1(\gamma|N), g_1(\gamma|N)]\), there are 3 equilibria, \{D, D\}, \{N, N\} and a mixed strategy. In this parameter region, \(\Pi_1^{NN} > \Pi_1^{DD}\) so we highlight the Pareto optimal equilibrium \{N, N\} as being the most likely.

**Proof of Corollary 1**

Corollary 1 follows immediately from Proposition 2, point 3.

**Sketch of Proof for Competitive Fringe equivalence to the main model**

When only Firm 1 can enter the foreign market, the gray marketer’s problem is to optimize equation (39) with respect to the choice of \(q_G\), which yields \(q_G = p_1 - p_F\). Substituting this \(q_G\) into Firm 1’s and Firm 2’s inverse demand functions in the decentralized and centralized organizational structures, respectively, and differentiating with respect to \(q_1, q_F\), and \(q_2\) yields identical first-order conditions for Firm 1 and Firm 2 as those in Appendix equations (ii) and (xi). The resulting optimal quantity choices and market prices are identical for each organizational structure, respectively. As a result, the optimal entry and organizational structure decisions are also identical.

When both Firm 1 and Firm 2 can enter the foreign market, the gray marketer’s problem is to optimize equation (40) with respect to the choice of \(q_{G1}\) and \(q_{G2}\), which yields \(q_{Gi} = p_i - p_{Fi}, i = 1, 2\). Substituting \(q_{G1}\) and \(q_{G2}\) into Firm 1’s and Firm 2’s inverse demand functions in the decentralized and centralized organizational structures, respectively, and differentiating with respect to \(q_1, q_{G1}, q_2,\) and \(q_{G2}\) yields identical first-order conditions for Firm 1 and Firm 2 as those in Appendix equations (vii) through (x), and (xiii) through (xvi), respectively. As in the one-firm case, the resulting optimal quantity choices and market prices are identical for each organizational structure, respectively. As a result, the optimal entry and organizational structure decisions are also identical.